"KamaChallenge-2015"

Mathematical Game "Penalty." Perm. October 31, 2015.

1. All natural numbers successively are written in a "spiral" in the cells of an infinite checkered plane (see pic). If we assume that in the conventional coordinate system the number 1 has coordinates (0;0), and the number 10 has coordinates (2; 1), then what are the coordinates of the number 2015?

	7	**8**	**9**	**10**
…	**6**	**1**	**2**	**11**
18	**5**	**4**	**3**	**12**
17	**16**	**15**	**14**	**13**

2. The length of the circle of the stadium is 400m. Three runners at the same time started a one-hour race from the same starting line; each of them has its own constant speed. The first runner run 20 km, the second one - 19 km, the third one – 18km. How many times during this race one of the runners outran another one?

3. Find the smallest natural number, which with added to its left side any nonzero digit *k* will be divisible by *k*.

4. What are the values of parameters *a*, *b* and *c* when the equation $ax^2+bx+c=0$ has exactly one solution?

5. Find the smallest number *a*, when in a square of side *a* could be located five circles of radius 1, which do not have common internal points pairwise.

6. Find the total number of different values which are taken by the function f(x)=[x]+[2x]+[3x] on the interval [0; 2015]. ([x] is the integer part of a number *x* and the maximum integer not greater than *x*)

7. Solve the inequality $\sqrt{\sin x}+\sqrt{\cos x}>1$

8. Present the number as the difference of two roots of neighboring natural numbers.

9. Diagonals of a convex quadrilateral *ABCD* are intersected at the point *O*. What is the least area can have this rectangle if the area ΔOAB is equal to 4 cm^2, and the area of ΔCOD is equal to 9 cm^2?

10. Find the sum of fractions $\frac{1}{1\cdot 2\cdot 3}+\frac{1}{2\cdot 3\cdot 4}+\frac{1}{3\cdot 4\cdot 5}+\ldots+\frac{1}{98\cdot 99\cdot 100}$.

Give the answer as an irreducible fraction.

11. Triangular pyramid is cut along its side edges and made a reamer. It turned out to be a square with a side of 1. Find the volume of the original pyramid.

12. In how many ways the set {1, 2, ..., 2015} can be divided into three non-empty sets, none of which contains any pair of consecutive positive numbers?

13. *AD* and *BC* are the bases of the trapezoid *ABCD* and they are equal to *a* and *b*. Diagonals cut off a segment from the middle line of a trapezoid. Find the length of the segment.

14. Three points with coordinates (0; 0), (0; 1) and (1; 0) are marked on the plane. It is allowed to mark new points, obtained by the symmetry of the marked points relative to the other marked points. What points can be marked?

15. Solve the system of equations: $\begin{cases} x = y(y-4) \\ y = x(x-4) \end{cases}$.

16. Place 32 chess knights on a Chess Board so that each of them takes exactly the other two.